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CROSSFLOW DEVELOPMENT IN THE BOUNDARY LAYER DURING LONGITUDINAL FLOW AROUND A RIGHT DIHEDRAL ANGLE

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UDC 532.526

Cases of the interaction of both laminar and turbulent boundary layers are realized in the flow around angular configurations. Investigations [1, 2] executed earlier indicate that during the interaction of turbulent boundary layers in the neighborhood of a bisectorial plane of a corner, crossflows in the form of counter-rotating vortex pairs develop.

This paper is devoted to an experimental investigation of the conditions for the origination and development of crossflows in the domain of boundary-layer interaction during the transition from the laminar to the turbulent state.

The tests were performed in the low-turbulence T-324 wind tunnel of the Institute of Theoretical and Applied Mechanics of the Siberian Branch of the USSR Academy of Sciences [3] under conditions of gradient-free flow around a right dihedral angle model. The description of the model construction and the fundamental measurement methodology are elucidated in [2]. The experiments were conducted at a mean stream velocity of $u_\infty = 7.6$ m/sec and a Reynolds number $\mathrm{Re}_1 \simeq 0.53 \cdot 10^6$, m^{-1}, initial degree of turbulence $\varepsilon \simeq 0.03\%$, and zero radius of conjugation of the angle faces in the working section. A constant temperature thermoanemometer 55D00 of the firm DISA in connection with a 55D10 linearizer was used as recording apparatus in measuring the longitudinal velocity component and its pulsations. A transducer with 0.65-mm-long and 3-μm-diameter Wollaston wire was used in the majority of tests, which assured a sufficiently low time constant τ. The working frequency band hence exceeded 40 kHz. All this permitted obtaining an acceptable resolution of the thermoanemometer system as a whole under the investigated conditions. The spectral characteristics of the velocity pulsations were investigated by using a frequency analyzer of 2010 type of the firm of Brüel and Kjer. The experience accumulated in the Institute of Theoretical and Applied Mechanics and other organizations [4-9], was used in measuring the velocity pulsations with the thermoanemometer.

The results of investigating the laminar-into-turbulent boundary-layer transitions during the flow around a right dihedral angle at supersonic speeds [10] showed that the boundary layer in the bisectorial plane becomes turbulent directly from the leading edge although laminar boundary layers are realized and interact on a certain section outside the domain of interaction on the faces of the angle.

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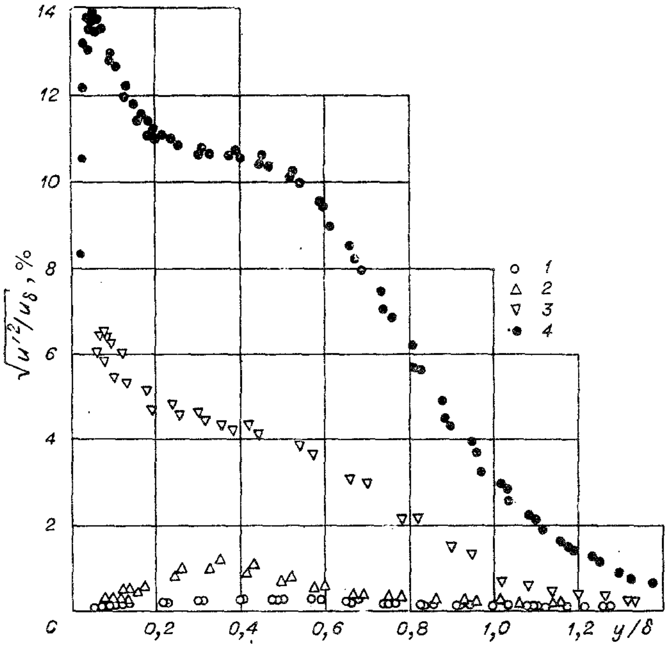

Fig. 1

For a more detailed study of this circumstance, measurements were made of the distribution of rms velocity pulsations in the boundary layer's several sections during removal from the model leading edge. Such measurements were executed both outside the domain of interaction and in the bisectorial plane of the angle ($y_1 = z_1$).

Profiles of the rms pulsations of the longitudinal velocity component measured in the boundary layer of the angle at a spacing of $z_1 = 62$ mm from the line of intersection of the planes, i.e., outside the interaction domain, are represented in Fig. 1 in the form of the dependence $\sqrt{u'^2}/u_\delta = f(y/\delta)$. Here u_δ is the velocity on the outer boundary of the boundary layer, y is a coordinate directed across the boundary layer in parallel to the bisectorial plane of the angle, δ is the boundary layer thickness determined under the condition that $u = 0.99\, u_\delta$. Points 1–4 denote the spacing x from the model leading edge, which respectively equals 389, 500, 700 and 890 mm.

It is seen that the boundary layer in the section x = 389 mm is laminar. Indeed, in this case the level of the rms pulsations is very low and due mainly to noise in the electronic equipment and to the specifics of its adjustment. Moreover, the mean velocity profile even agrees satisfactorily with the well-known Blasius theoretical profile. The magnitude of the pulsations grows with distance from the leading edge and reaches the maximum value, which is about 14%, near the wall. Estimates show that in this case the pulsation distribution over the height of the boundary layer differs insignificantly from an analogous dependence, which is valid in the domain of developed turbulent flat-plate flow. This indicates that under the conditions investigated the boundary layer at $Re_x \simeq 0.47 \cdot 10^6$ is almost in equilibrium.

It is characteristic that independently of the state of the interacting boundary layer, the boundary layer in the bisectorial plane of the angle itself is already turbulent for $Re_x \simeq 2 \cdot 10^5$. This is indicated by Fig. 2 in which profiles of the rms pulsations of the longitudinal velocity component are represented for $y_1 = z_1$ (the same notation as in Fig. 1). It is seen that the pulsation level is sufficiently high even in the first section (x = 389 mm) and is around 9% in the near wall flow domain. The quantity $\sqrt{u'^2}/u_\delta$ varies insignificantly along the length of the model, which indicates a turbulent boundary layer state. This is also confirmed by the character of the distribution of the "power spectrum density" function obtained in both the section x = 389 mm and at higher values of the coordinate x. However, as the leading edge of the angle is approached, the pulsation level is noticeably reduced, and individual discrete frequencies are observed on the spectrograms. To clarify the reason for this phenomenon, additional measurements were made in order to determine the location of the laminar-to-turbulent transition domain. They showed that the turbulent boundary layer in the bisectorial plane of the angle is not established from the leading edge itself for $Re_1 = 0.53 \cdot 10^6, m^{-1}$, but a certain transition zone exists (to $Re_x \simeq 1.7 \cdot 10^5$) which is apparently due to both the intersection of the leading edges of the faces executing the part of turbulizer, and to the nature of the boundary layer interaction on the initial part of the model. Therefore, the oscillations of a discrete frequency observed in the spectrograms are evidently analogous to those which occur

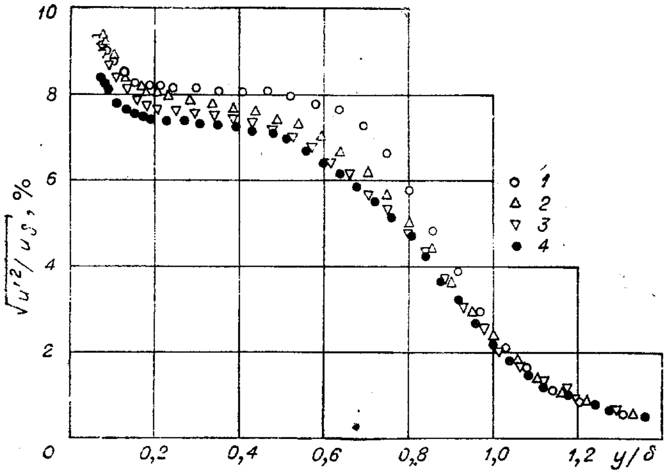

Fig. 2

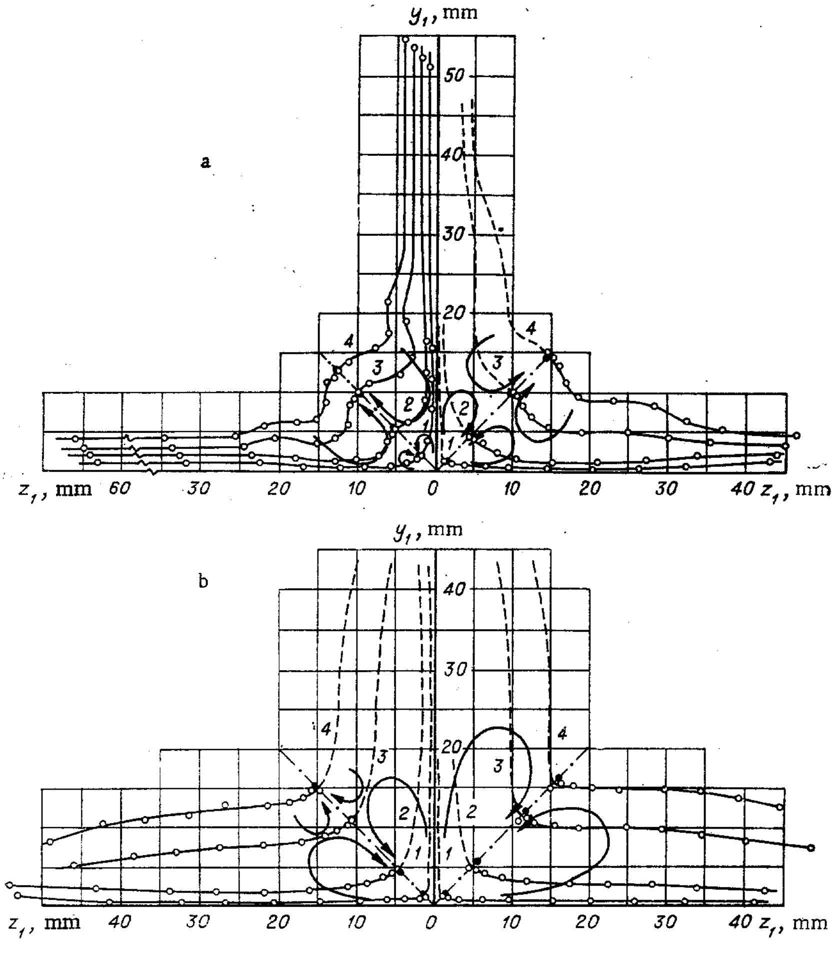

Fig. 3

in the pretransition and transition domains of a plane boundary layer [4]. This requires a special study. The results presented here show convincingly that a turbulent boundary layer is realized in the neighborhood of the bisectorial plane during interaction of the laminar boundary layers being developed on the faces of the angle, exactly as during the interaction of turbulent boundary layers downstream. Crossflows in the form of counter-rotating vortex pairs are hence clearly observed in the interaction domain which is broadened with the removal from the leading edge.

A typical distribution of the isotachs (equal velocity lines) in the sections where laminar and almost laminar boundary layers interact, is presented in Fig. 3a (to the left of the section x = 389 mm, and to the right of

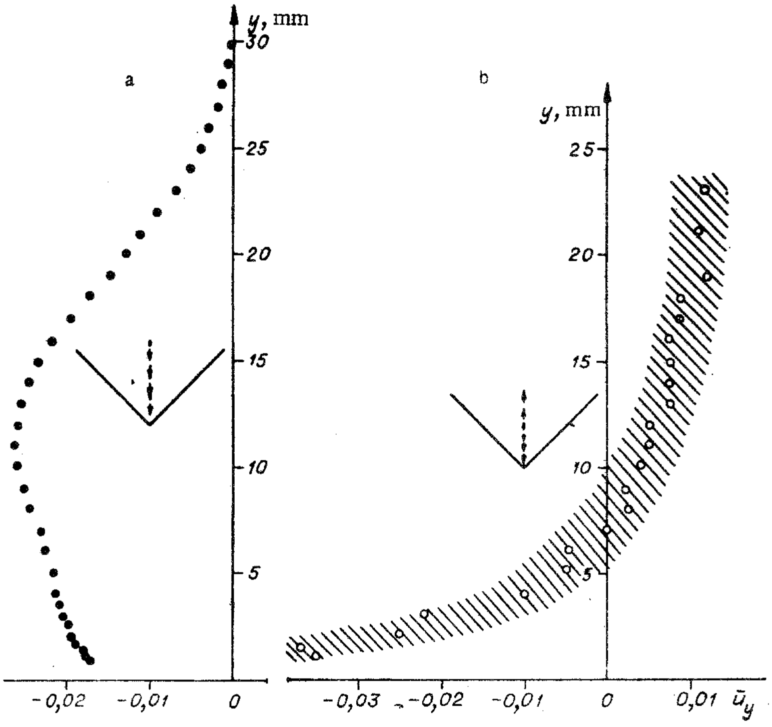

Fig. 4

the section $x=500$ mm). Here the isotachs 1-4 correspond to the velocity ratio $u/u_\delta=0.5$; 0.7; 0.9 and 0.99. The isotach contour in the neighborhood of the bisectorial plane is convex in these sections and the vortex rotation is towards the angular lines and from it to the external flow. Conversely, in the sections where the transition and turbulent boundary layers interact (Fig. 3b, to the left of the section $x=700$ mm and to the right of the section $x=890$ mm), the isotach contour is concave, and the vortex rotation in the section is along the bisectorial plane to the angular line and from it along the span of the angle. The open circles denote data obtained by a penumatic probe, and the dark circles by the thermoanemometer. In all cases the isotachs outside the domain of boundary layer interaction are parallel to the faces of the angle, while as has been shown earlier [2], the velocity profile is the same as in the flow around a flat plate. It can be assumed that the process of freezing the gas near an angular line plays a definite part in formation of the isotach contour. Nevertheless, the final picture of the isotach distribution during laminar and turbulent boundary layer interaction is substantially different. Therefore, distortion of the isotach contour is determined mainly by the occurring crossflows. It hence remains unclear how inversion of the rotation of the vortices being developed in the neighborhood of the bisectorial plane occurs as the interacting laminar boundary layers make the transition into turbulent layers.

In order to clarify this circumstance, profiles of the transverse velocity component were measured in the bisectorial plane of the angle during the interaction of both the laminar and turbulent boundary layers. A profile of the component $\bar{u}_y=u_y/u_\delta$, obtained during the interaction of developed turbulent flows, is shown in Fig. 4a. It is seen that the quantity $\bar{u}_y$ is negative in practically the whole height of the layer. This means that the crossflows in the bisectorial plane are directed from the outer boundary of the boundary layer towards the angular line. On the other hand, the profile of the transverse component obtained during laminar boundary-layer interaction (Fig. 4b) affords the foundation to consider that cross flows directed to the angular line and the outer boundary of the boundary layer occur in the bisectorial plane of the angle. Indeed, the transverse component $\bar{u}_y$ has both positive and negative values in this case. It should be noted that this profile is obtained with great error because of the very small values of the transverse velocity component (less than 3% of u_δ), and can be considered only for qualitative estimates. The probable data-spread band, shown in Fig. 4b, affords a foundation to assume that motion towards the angular line near the wall occurs during laminar boundary-layer interaction ($x=389$ mm) together with the predominant motion from the angular line to the upper half of the layer.

Starting from the above, a scheme can be represented for crossflow development in the domain of boundary-layer interaction along the model length. Their direction is shown provisionally by arrows in the background of the equal-velocity lines (see Figs. 3a and b). It is evident that not one but two or possibly even a greater quantity of vortices is realized on each side of the bisectorial plane of the angle during laminar bound-

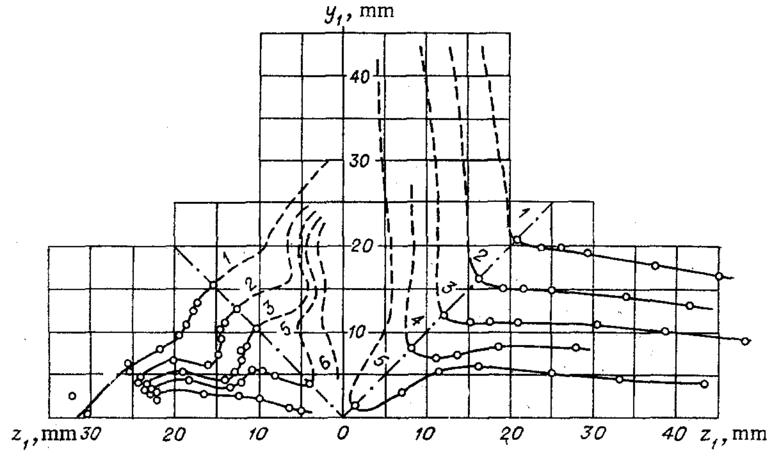

Fig. 5

ary-layer interaction (see Fig. 3a to the left). Indeed, taking into account the symmetric nature of the flow relative to the bisectorial plane of the angle, this latter can provisionally be replaced by some solid boundary. Then it is conceivable that the crossflow directed approximately along the normal to this boundary will attach to it and being spread to the outer boundary of the boundary layer and to the angular line, will form two counter-rotating vortices. The upper vortex is evidently predominant during laminar boundary-layer interaction, hence the corresponding deformation of the isotachs toward the external flow. The lower vortex gradually increases with distance from the model leading edge, and replaces the upper vortex, which apparently vanishes in the long run because of the effects of viscous dissipation in the boundary layer and the diminishing influence of the laminar sublayer. For $x=890$ mm (see Fig. 3b to the right) just one vortex is evidently realized which has the direction of rotation towards the angular line, as is confirmed by the profile of the transverse velocity component in Fig. 4a. Therefore, the direction of vortex rotation during turbulent boundary-layer interaction is opposite to that which was observed during laminar boundary-layer interaction.

The feature noted is also verified by the equal pulsation lines of the longitudinal velocity component $\sqrt{u'^2}/u_\delta = \text{const}$, which are presented on the left and right, respectively, in Fig. 5. Curve 1 here corresponds to the magnitude of the pulsation $\sqrt{u'^2}/u_\delta = 0.01$, 2) 0.03, 3) 0.06, 4) 0.074, 5) 0.08, 6) 0.10. It is seen that depending on the distance from the leading edge the lines of equal velocity pulsations are deformed, as are the isotachs, to either the outer boundary of the boundary layer ($x=391$ mm) or to the angular line ($x=897$ mm), thereby reflecting the crossflow specifics. Moreover, the pulsation profiles across the boundary layer at $x=897$ mm show that their level in the bisectorial plane of the angle is substantially below that outside the interaction domain. This is caused by the fact that a gas with a low degree of turbulence is carried over by the crossflows from the external flow towards the angular line. This latter naturally results in a reduction in the magnitude of the pulsations in the plane of symmetry of the angle and confirms the deduction that the crossflows during turbulent boundary layer interaction are directed along the bisectorial plane to the angular line.

Therefore, it has been shown experimentally that secondary flows in the form of vortices with mutually opposing directions of rotation are developed in the laminar boundary-layer interaction domain in the flow around a right dihedral angle with zero radius of conjugation of the faces. Still another pair of vortices with opposite direction of rotation is generated in the neighborhood of the angular line in addition to this predominant crossflow. A smooth inversion of the direction of rotation of the predominant vortex pairs occurs as the interacting laminar boundary layers make the transition into turbulent layers so that the crossflows in the zone of turbulent boundary-layer interaction are directed towards the angular line and from it along the span of the faces.

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THEORY OF A FLAT SUBMERGED JET OF A NON-NEWTONIAN LIQUID WITH A POWER RHEOLOGICAL LAW

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The character of the propagation of shear perturbations in non-Newtonian liquids with a power rheological law [1]

$$\sigma_{ij} = 2k\,(f_{\alpha\beta}f_{\alpha\beta})^{(n-1)/2}f_{ij} \tag{1}$$

is essentially determined by the value of the exponent n in (1), where σ_{ij} is the deviator of the stress tensor; f_{ij} is the tensor of the deformation rates; k and n are rheological constants of the medium. With the terminology adopted, media with $n>1$ are called dilatant, and with $n<1$ they are called pseudoplastic; the case $n=1$ corresponds to a Newtonian viscous liquid. It is well known that, in dilatant liquids, shear perturbations are propagated with a finite rate, whereas, in pseudoplastic and Newtonian viscous liquids, the rate of propagation of perturbations is infinite [2, 3]. As a result of this, there is a finite thickness of the boundary layer with laminar flow of a dilatant liquid past a flat semiinfinite plate. Actually, the finite thickness of the boundary layer in this case is explained by the fact that the shear perturbations, propagating with a finite velocity, are carried along the flow and emerge to the surface, at which the layer is formed only at a finite distance in the direction of its transverse coordinate. The inexact picture given in [4], unjustifiably excluded the fact of the finite thickness of the boundary layer in the case of "densifying" dilatant liquids with $1<n<2$. At the same time, the finite thickness of the boundary layer can be rigorously shown in the case of any given dilatant liquid with arbitrary values of $n>1$.

If, in dilatant liquids, the rate of propagation of shear perturbations is finite, a flat laminar jet immersed in such liquids should have a finite thickness, i.e., at a finite distance from the axis of the jet in the liquid there is a surface $y=y_{\Phi}(x)$ outside of which the longitudinal component of the velocity is equal to zero (see Fig. 1). This is connected with the fact that the jet brings into motion the liquid into which it flows out; in addition to the longitudinal, the liquid has a transverse component of the velocity, directed toward the axis of the jet. On the other hand, the rate of propagation of shear perturbations in dilatant liquids, connected with a change in the longitudinal component of the velocity, decreases with an increase in the distance from the source

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